SNUTP 95-050

hep-ph/9505400

Normalization of the Perturbative QCD Corrections for $B \to X_s \gamma$ Decay

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Abstract

We study the normalization of perturbative QCD corrections to the inclusive $B \to X_s\gamma$ decay. We propose to set the renormalization scale using the Brodsky-Lepage-Mackenzie (BLM) method. In the proposed method the scale is determined by absorbing the vacuum polarization correction from light fermions to renormalization scale but not including the anomalous dimensions. The BLM scale depends in general on the renormalization scheme and the factorization scale. We find that the BLM scale is insensitive to the factorization scale. In the heavy-quark potential scheme, we find that the BLM scale is $\mu_{BLM} \approx (0.315 - 0.334) m_b$ when the factorization scale varies from $m_b/2$ to $2m_b$.

12.38.Bx, 13.40.Hq, 11.10.Gh

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The rare decay process $b \to s\gamma$ has been recently observed at CLEO II [1] at a branching ratio $2.32(\pm 0.57 \pm 0.35) \times 10^{-4}$ (statistical and systematic errors). The process serves as an excellent probe to the new physics beyond the Standard Model such as charged Higgs [2], supersymmetry [3] or anomalous $WW\gamma$ coupling [4]. At leading order the process is described by an electromagnetic penguin diagram. It is known that there are large QCD corrections to the penguin diagram. A complete renormalization group-improved perturbative calculation is now available to the leading-logarithm approximation. It is found that the QCD corrections increase the rate by a factor two to three. In the Standard Model the branching ratio to leading-logarithmic accuracy is $2.8(\pm 0.8) \times 10^{-4}$, where the error is dominated by the uncertainty in the QCD renormalization scale varied over $m_b/2 < \mu < 2m_b$ [5,6]. Inclusion of the next-to-leading logarithmic QCD corrections will reduce the theoretical error. However, a complete calculation is still not achieved. With partial next-to-leading corrections known so far, the branching ratio falls to $\sim 1.9 \times 10^{-4}$.

In view of the current status of the theoretical uncertainties as summarized above it is of interest to normalize the leading order QCD corrections and estimate theoretical errors thereof. In this paper we address this issue and determine the renormalization scale according to the Brodsky-Lepage-Mackenzie (BLM) method [7]. So far applications of the BLM method have been mostly restricted to processes involving (partially) conserved currents. In this case QCD correction may be calculated by a perturbative expansion at a fixed order. For example, BLM scale setting has been established for the inclusive semileptonic B decay $B \to X_q e\overline{\nu}$ [8]. In Ref. [8] the BLM scale in the $\overline{MS}$ scheme is found $\mu_{BLM} \approx 0.07\, m_b$ ($0.12\, m_b$ when running $\overline{MS}$ b-quark mass [9] is used), indicating a significant QCD correction from higher order terms. The QCD corrections for the radiative B decay is more complicated since the process involves large logarithms arising from QCD effects between the top-quark or W-gauge boson mass and the bottom quark mass scales. The effective Hamiltonian approach takes care of these large logarithms by introducing a factorization scale and separates the long- and the short-distance QCD effects.

In situations involving large logarithms a physical observable P such as the total decay

rate or the branching ratio may be written schematically as

$$P = M(\mu)\Big[r_0(\mu) + r_1(\mu,\mu_R)\frac{\alpha_s(\mu_R)}{\pi} + n_f r_2(\mu,\mu_R)\Big(\frac{\alpha_s(\mu_R)}{\pi}\Big)^2 + \cdots\Big]. \tag{1}$$

Here μ is a factorization scale dividing long- and short-distance physics and μ_R is a renormalization scale for the QCD corrections. All the large logarithms of (m_W/μ) are resummed into $M(\mu)$. QCD corrections to the matrix element at $\mu, \mu_R \gg \Lambda_{QCD}$ are given by a perturbative expansion. The coefficients r_0, r_1 and r_2 depend on both the factorization and the renormalization scales. Of course the physical observable P calculated to all orders should be independent of the two scales. However, at any finite order, perturbative evaluation of P do depend on the factorization and the renormalization scales. In practice the factorization scale is set equal to the renormalization scale, and then the theoretical error is estimated. To be precise these two scales are distinct, and there is no reason why the two scales should be set equal. For example, in the conformally invariant QCD, a finite-order perturbatiion expansion does not depend on any renormalization scale but still depends explicitly on the factorization scale.

We are interested in calculating perturbative QCD corrections and setting the renormalization scale μ_R for a given renormalization scheme and the factorization scale μ. Typically there are logarithms of (μ_R/μ) present from two sources. One is the logarithms that can be resummed to the running coupling constants. Another is the logarithms that may be resummed into the anomalous dimensions of operators involved. In order to set the renormalization scale we propose to follow the original spirit of BLM closely. The idea is to set μ_R by absorbing $n_f\alpha_s^2$ contributions of the vacuum polarization to the renormalization scale, but not including the contributions of the anomalous dimensions. Then the generalized BLM scale is set by the fixed-order perturbative correction part inside the bracket in Eq.(1): $\mu_{BLM} = \mu_R \exp(3r_2/r_1)$. If the content of the theory does not change between μ and μ_R, the normalized physical quantity $\overline{P}$ may be given by

$$\overline{P} = M(\mu_{\rm BLM})\Big[r_0(\mu_{BLM}) + r_1(\mu_{BLM})\frac{\alpha_s(\mu_{\rm BLM})}{\pi}\Big] \tag{2}$$

in which the anomalous dimension effects are taken into account by running from $M(\mu)$ to $M(\mu_{BLM})$ using the renormalization group equation. Aside from setting the BLM scale it may be also argued that the fixed order coefficients r_1, r_2 are the most significant parts in the next-to-leading order contribution [6,11].

We emphasize, however, that our proposal is by no means unique or superior to other plausible generalizations of the BLM method. For example, we may set the BLM scale including contributions from the next-to-leading order anomalous dimensions as well as effective coupling constant through the vacuum polarization. This is complicated and in order to be consistent, we need anomalous dimensions at next-to-leading order. Furthermore at present a complete next-to-leading order calculation of the coefficient functions and anomalous dimensions is not available for the radiative B decay. On the other hand our method provides a physically motivated generalization of the original BLM method and the final result shows a reasonable behavior.

For the radiative B decay, the effective Hamiltonian below M_W scale is given by [10]

$$H_{\rm eff} = -V_{tb}V_{ts}^{*}\frac{G_F}{\sqrt{2}}\sum_{i=1}^{8} C_i(\mu)\,\mathcal{O}_i(\mu), \tag{3}$$

where μ is the factorization scale separating the coefficient functions and the operators. For the partonic decays $b \to s\gamma$ and $b \to s\gamma g$, the dominant contributions in the radiative decay arise from the operators [10]

$$\begin{aligned}
\mathcal{O}_1 &= (\overline{c}_{L\alpha}\gamma^{\mu} b_{L\beta})(\overline{s}_{L\beta}\gamma_{\mu} c_{L\alpha}) \\
\mathcal{O}_2 &= (\overline{c}_{L\alpha}\gamma^{\mu} b_{L\alpha})(\overline{s}_{L\beta}\gamma_{\mu} c_{L\beta}) \\
\mathcal{O}_7 &= \frac{e}{16\pi^2} F_{\mu\nu}\overline{s}_{\alpha}\sigma^{\mu\nu}(m_b P_R + m_s P_L) b_{\alpha} \\
\mathcal{O}_8 &= \frac{g_s}{16\pi^2} G^{A}_{\mu\nu}\overline{s}_{\alpha}\sigma^{\mu\nu}T^{A}_{\alpha\beta}(m_b P_R + m_s P_L) b_{\beta}
\end{aligned} \tag{4}$$

where α, β are color indices, $P_{R,L} = (1 \pm \gamma_5)/2$ and $F_{\mu\nu}, G^{A}_{\mu\nu}$ are photon and gluon field strengths respectively. Contributions of the other operators enter through operator mixing

at next-to-leading order and are numerically negligible [11]. Because of color structure, $\mathcal{O}_1$ does not contribute to the real gluon emission to the order we calculate. In addition $C_8(m_b)$ is small compared to the others. Hence we neglect $\mathcal{O}_1$, $\mathcal{O}_8$ contributions in what follows.

The Wilson coefficients at leading order are given by [10]

$$\begin{aligned} C_2(\mu) &= \frac{1}{2}\Big[\eta^{-6/23} + \eta^{12/23}\Big]C_2(m_W), \\ C_7(\mu) &= \eta^{-16/23}\Big\{C_7(m_W) - \frac{58}{135}\Big[\eta^{10/23} - 1\Big]C_2(m_W) \\ &\quad - \frac{29}{189}\Big[\eta^{28/23} - 1\Big]C_2(m_W)\Big\}, \end{aligned} \tag{5}$$

with $\eta = \alpha_s(\mu)/\alpha_s(m_W)$ and

$$\begin{aligned} C_2(m_W) &= 1, \\ C_7(m_W) &= \frac{x}{24(x-1)^4}[6x(3x-2)\ln x \\ &\quad - (x-1)(8x^2+5x-7)], \end{aligned} \tag{6}$$

with $x = m_t^2/m_W^2$.

Keeping only $\mathcal{O}_2$ and $\mathcal{O}_7$ the total $b \to s\gamma$ decay rate can be written as

$$\Gamma = \Gamma_{77} + \Gamma_{22} + \Gamma_{27}, \tag{7}$$

where $\Gamma_{22}(\Gamma_{77})$ denotes the decay rate obtained from the matrix elements squared for $\mathcal{O}_2$ ($\mathcal{O}_7$) and Γ_{27} is the decay rate from the interference terms between $\mathcal{O}_2$ and $\mathcal{O}_7$. Including both virtual correction and real gluon emission Γ_{77} is given by [11]

$$\Gamma_{77} = \Gamma_0\, C_7(\mu)^2 \Big[1 - \frac{2\alpha_s(\mu_R)}{3\pi}(\frac{2\pi^2}{3} - \frac{8}{3}) + \cdots\Big], \tag{8}$$

where $\Gamma_0 = |\lambda_t|^2 \alpha G_F^2 m_b^5/(32\pi^4)$ and $\lambda_t = V_{tb}V_{ts}^*$. Here we neglect the s quark mass and the ellipsis denotes higher order contributions in α_s.

To set the generalized BLM scale according to our proposal, it is necessary to calculate the QCD corrections of order α_s^2, which are proportional to the number n_f of the light quark flavors. Smith and Voloshin [12] have recently shown that the n_f-dependent part of

the order α_s^2 contribution may be written in terms of the one-loop corrections evaluated with a fictitious gluon mass λ

$$\delta\Gamma^{(2)} = -\frac{b\alpha_s^{(V)}(m_b)}{4\pi} \times \int_0^\infty \Big[\Gamma^{(1)}(\lambda) - \frac{m_b^2}{\lambda^2+m_b^2}\Gamma^{(1)}(0)\Big]\frac{d\lambda^2}{\lambda^2}, \tag{9}$$

where $b = 11 - 2n_f/3$ and $\Gamma^{(1)}(\lambda)$ is the order α_s contribution to the decay rate computed with a finite gluon mass λ and $\alpha_s^{(V)}(m_b)$ is the QCD coupling obtained in the heavy-quark potential scheme.

In order to obtain Γ_{77} we calculate the virtual and bremsstrahlung corrections with a finite gluon mass coming from $\mathcal{O}_7$. The virtual massive gluon correction is obtained by calculating diagrams corresponding to the dressing of the vertex, the initial-state b quark and the final-state s quark line with a massive gluon. The bremsstrahlung correction is calculated from the emission of a massive gluon from either of the external b, s quarks. Sum of the two contributions are infrared finite. The sum is calculated analytically as

$$\begin{aligned}\Gamma_{77}^{(1)}(x) = \frac{\alpha_s}{3\pi}\Gamma_0\Big[&\frac{16}{3} - \frac{4}{3}\pi^2 \\ &- 16x - \frac{1}{2}x^2 + \frac{2}{3}x^3 - 3x^2\ln x \\ &+ \frac{x(3x^2-10x-20)}{\sqrt{x(4-x)}}\tan^{-1}\frac{(1-x)\sqrt{x(4-x)}}{x(3-x)} \\ &+ \frac{x(28+2x-3x^2)}{\sqrt{x(4-x)}}\tan^{-1}\sqrt{\frac{4-x}{x}} \\ &+ 8\tan^{-1}\frac{\sqrt{x(4-x)}}{2-x}\tan^{-1}\sqrt{\frac{4-x}{x}}\Big],\end{aligned} \tag{10}$$

where x is the dimensionless gluon to b quark mass ratio, $x \equiv \lambda^2/m_b^2$ and Γ_0 is the tree level decay width.

After carrying out the integration of Eq. (10) over x according to Eq. (9), Γ_{77} at order α_s^2 is given by

$$\begin{aligned}\Gamma_{77} = (C_7(\mu))^2\Gamma_0\Big[1 - &\frac{4\alpha_s^V(m_b)}{3\pi}(\frac{\pi^2}{3} - \frac{4}{3}) \\ &\times \Big(1 + \frac{b\alpha_s^V(m_b)}{4\pi}[2.733]\Big)\Big].\end{aligned} \tag{11}$$

We can apply the same method for the calculation of Γ_{22} and Γ_{27}. There are no virtual corrections at order α_s. The double differential decay rates at order α_s with a finite gluon mass are given by

$$
\begin{aligned}
\frac{d\Gamma_{22}}{dx_q dx_\gamma} &= \frac{2\alpha_s}{27\pi}\Gamma_0 (C_2(\mu))^2 \times \Big[2\,(1-A)^2 \Big(x_g(1-x-x_\gamma) + x x_q \Big) \\
&- 2(1-A^2)\Big\{ x_\gamma(1-x-x_\gamma) + x_g(1+x-x_g) + x_q(1-x-x_q)\Big\} \\
&+ (1+A)^2 \Big\{ x_\gamma(1+x-x_g) + \frac{1-x-x_q}{2x}(x_\gamma(1-x-x_\gamma) + x_g(1+x-x_g))\Big\} \\
&+ \Big\{ -1 - 2A + 3A^2 + \frac{8(1-A)x}{1-x-x_q} - \frac{4(1-A)^2 x^2}{(1-x-x_q)^2}\Big\} \\
&\times \Big\{ -x_\gamma(1+x-x_g) + \frac{1-x-x_q}{2x}(x_\gamma(1-x-x_\gamma) + x_g(1+x-x_g))\Big\}\Big], \qquad (12)
\end{aligned}
$$

where $x_i = 2E_i/m_b$ ($i = q, \gamma, g$ are s quark, photon, gluon in the final state) satisfying $x_q + x_\gamma + x_g = 2$ and $x = \lambda^2/m_b^2$. And

$$
A = \frac{x}{1-x-x_q} \ln \frac{1-x_q}{x}. \qquad (13)
$$

Similarly

$$
\begin{aligned}
\frac{d\Gamma_{27}}{dx_q dx_\gamma} &= -\frac{\alpha_s}{9\pi}\Gamma_0 C_2(\mu) C_7(\mu) \\
&\times \Big[\frac{1}{x-x_g}\Big\{ 2(1+x-x_g)(1-x-x_q) + x_\gamma(1-x-x_\gamma) + \frac{2x x_\gamma(1+x-x_g)}{1-x-x_q} \\
&- x_q(1-x-x_q) - x_g(1+x-x_g) - 4x(1+x-x_g) \\
&+ A\Big((x_g - 4x_\gamma)(1+x-x_g) + (4(1+x-x_g) + x_q)(1-x-x_q) - x_\gamma(1-x-x_\gamma) \\
&+ \frac{2x_g}{x}(1+x-x_g)(1-x-x_q) - \frac{2x x_\gamma(1+x-x_g)}{1-x-x_q}\Big)\Big\} \\
&+ \frac{1}{1-x_\gamma}\Big\{ x_q(1-x-x_q) + x_\gamma(1-x-x_\gamma) - x_g(1+x-x_g) + 2x x_\gamma \frac{1+x-x_g}{1-x-x_q} \\
&+ A\Big(-2x x_\gamma \frac{1+x-x_g}{1-x-x_q} + \frac{1-x-x_q}{x}(x_g(1+x-x_g) + x_\gamma(1-x-x_\gamma) - x_q(1-x-x_q)) \\
&- 4x_\gamma(1+x-x_g) - x_q(1-x-x_q) - x_\gamma(1-x-x_\gamma) + x_g(1+r-x_g)\Big)\Big\}\Big]. \qquad (14)
\end{aligned}
$$

These are infrared finite as $x \to 0$ and the phase-space integration and the integration over the gluon mass yield finite results. We can get the single differential decay rates by integrating Eqs. (12) and (14) over, say x_q, analytically and the remainingg calculation is done numerically. Using the Eq. (9) and after the integration, we find

$$\Gamma_{22} = +C_2^2\Gamma_0\frac{4\alpha_s^V(m_b)}{81\pi}\Big[1+\frac{b\alpha_s^V(m_b)}{4\pi}[2.486]+\cdots\Big], \tag{15}$$

$$\Gamma_{27} = -C_2C_7\Gamma_0\frac{2\alpha_s^V(m_b)}{27\pi}\Big[1+\frac{b\alpha_s^V(m_b)}{4\pi}[6.648]+\cdots\Big]. \tag{16}$$

Summing Eqs. (11), (15), and (16) the total decay rate is given by

$$\begin{aligned}\Gamma(\mu) &= \Gamma_{77}(\mu)+\Gamma_{22}(\mu)+\Gamma_{27}(\mu)\\ &= \Gamma_0(C_7(\mu))^2\Big[1+\frac{\alpha_s}{3\pi}\Big(-7.83-0.222\kappa+0.148\kappa^2\Big)\\ &\times\Big(1+\frac{b\alpha_s}{4\pi}\frac{21.5+1.48\kappa-0.368\kappa^2}{7.84+0.222\kappa-0.148\kappa^2}\Big)+\cdots\Big],\end{aligned} \tag{17}$$

where $\kappa = C_2/C_7$ is the ratio of the Wilson coefficient functions. In Eq. (17) the terms proportional to n_f can be absorbed into the order α_s term according to the BLM prescription. Recall that there are two independent scales in the decay rate. One is the factorization scale μ at which the Wilson coefficient functions and the local operators are factorized and the other is the renormalization scale μ_R at which the theory is renormalized, and the two scales are independent.

As we have mentioned, we propose to set the BLM scale only from the fixed-order perturbation series in $\alpha_s(\mu_R)$. This method extends the original idea of Brodsky, Lepage and Mackenzie [7] closely in the sense that the BLM scale is determined by absorbing the vacuum polarizations of light fermions into the coupling constant $\alpha_s(\mu_{BLM})$. In this case the coefficients in front of the powers of $\alpha_s(\mu_R)$ are functions of the factorization scale through the anomalous dimension of the operators. Therefore the BLM scale depends on the factorization scale. Noting that in the heavy quark potential scheme

$$\alpha_s^{(V)}(m_b) = \alpha_s^{(V)}(Q)(1+\frac{b}{4\pi}\alpha_s^{(V)}(Q)\ln\frac{Q^2}{m_b^2}), \tag{18}$$

the BLM scale from Eq. (17) is given by

$$\mu_{BLM} = m_b\exp\Big[-\frac{1}{2}\cdot\frac{21.4+1.48\kappa-0.368\kappa^2}{7.83+0.222\kappa-0.148\kappa^2}\Big]. \tag{19}$$

Here κ is a function of the factorization scale μ. In Fig. 1, the BLM scale μ_{BLM} is plotted as a function of the factorization scale μ. For numerical evaluation we have used $m_t = 180$

GeV, $m_W = 80.1$ GeV, $m_b = 5$ GeV and $\Lambda = 225$ MeV for $n_f = 5$. Our result in Fig. 1 shows that the BLM scale is extremely insensitive to the factorization scale. As we vary the factorization scale μ from $m_b/2$ to $2m_b$, the BLM scale changes only over $0.315\, m_b$ to $0.334\, m_b$. This is gratifying since the insensitivity of the factorization scale implies that we can reduce part of the theoretical error of the decay rate from the choice of the factorization scale. In our case the renormalization scale is almost constant and it yields the normalized decay rate which does not vary widely when the factorization scale changes from $m_b/2$ to m_b.

The total decay rate is then normalized by scaling down the leading order coefficient functions from μ to $\mu_{\rm BLM}$ using the leading logarithmic coefficient function. According to the original idea of BLM, the normalized rate may be interpreted as the mean total decay rate averaged over the gluon virtuality. In order to compare this normalized total decay rate with other decay rates, we have plotted three decay rates as a function of the factorization scale in Fig. 2. The dotted line represents the α_s^0-order result from $\mathcal{O}_7$ only. The dashed line is the α_s order result in Eq. (17) with the factorization scale μ and the solid line is the α_s result with $\mu = \mu_{BLM}(\mu)$. As shown in Fig. 2, the result with the BLM scale is almost flat in the range while other results vary. Note that in plotting Fig. 2, we have used the coefficient functions only to the leading logarithm order. If the complete next-to-leading logarithmic results for the coefficient functions and the anomalous dimensions are available in the future, it will be interesting to compare the next-to-leading log result of the decay rate with our BLM result which will also be improved by using the coefficient functions at next-to-leading orer. This will eventually test whether our proposal of setting the renormalization scale is sensible.

We can also ask questions like how the BLM scales and the decay rate can be expressed in other schemes. Since there exists a definite relationship of μ_{BLM} and physical observables among various schemes, our result can be converted without any ambiguity to other schemes such as the $\overline{MS}$ scheme and can also be extended to include the running mass effects of the b quark. [15]

We thank S. Brodsky, L. Dixon, C. Greub, M. Lautenbacher and M.B. Wise for many helpful discussions. This work was supported in part by KOSEF Grant 941-0200-02202 (JGC), U.S. NSF-KOSEF Bilateral Grant (SJR), KRF Nondirected Grant '94 (SJR), Ministry of Education SNU-BSRI 94-2418 (SJR), KU-BSRI 94-2408 (JGC) and KOSEF SRC Program (JGC, SJR).

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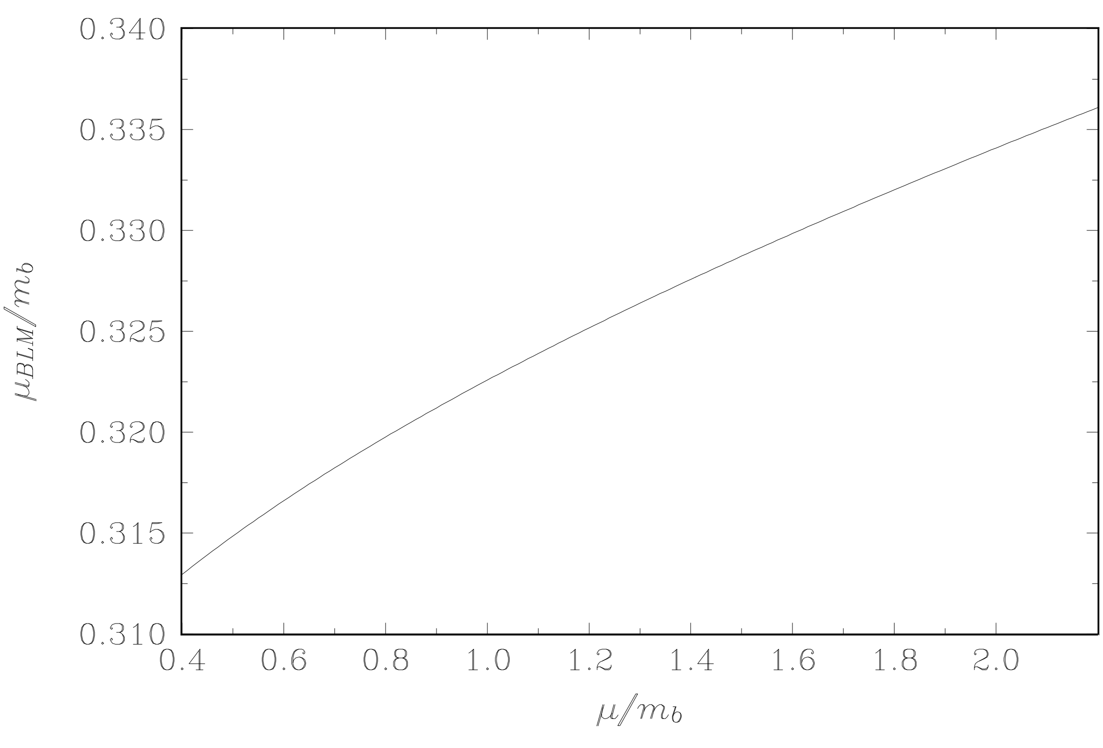
FIGURES

FIG. 1. The BLM scale μ_{BLM}/m_b for the total decay rate Γ as a function of μ/m_b.

FIG. 2. The total decay rates Γ/Γ_0 as a function of μ/m_b.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0.335
0.330
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0.320
0.315
0.310
0.4
0.6
0.8
1.0
1.2
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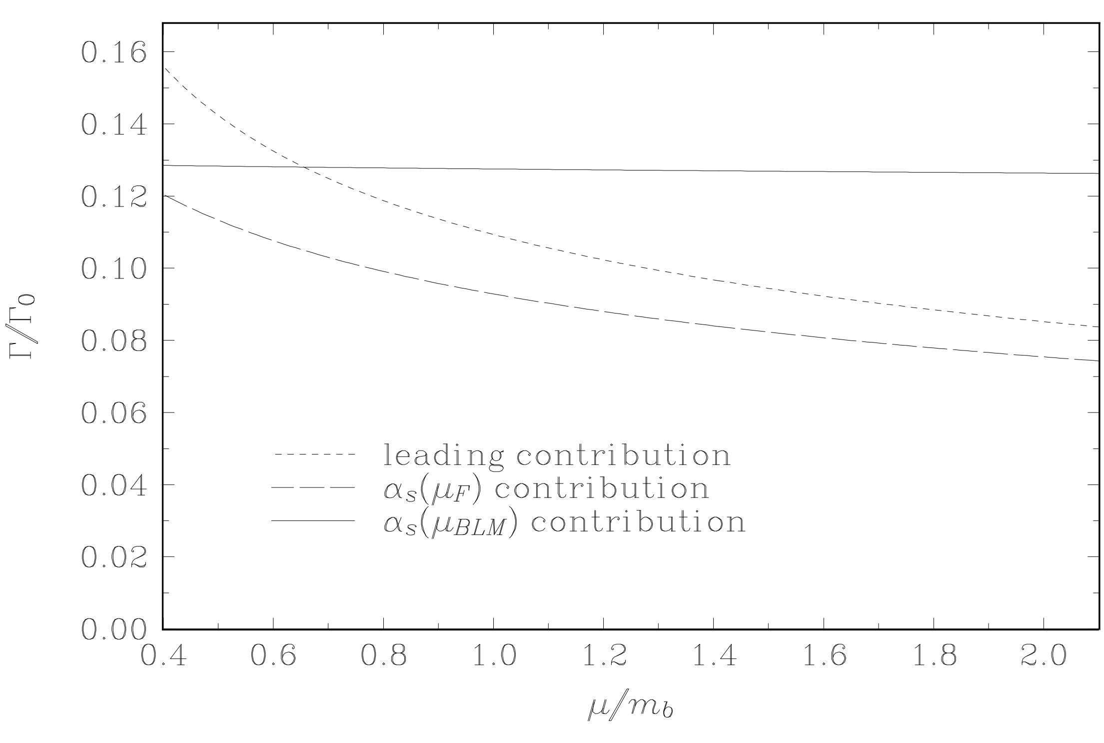
1.6
1.8
2.0
μ_{BLM}/m_b
μ/m_b



0.16
0.14
0.12
0.10
0.08
0.06
0.04
0.02
0.00
Γ/Γ_0
0.4
0.6
0.8
1.0
1.2
1.4
1.6
1.8
2.0
μ/m_b
leading contribution
$\alpha_s(\mu_F)$ contribution
$\alpha_s(\mu_{BLM})$ contribution